Displacement Detection with a Vibrating RF SQUID: Beating the Standard Linear Limit

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We study a novel configuration for displacement detection consisting of a nanomechanical resonator coupled to both, a radio frequency superconducting interference device (RF SQUID) and to a superconducting stripline resonator. We employ an adiabatic approximation and rotating wave approximation and calculate the displacement sensitivity. We study the performance of such a displacement detector when the stripline resonator is driven into a region of nonlinear oscillations. In this region the system exhibits noise squeezing in the output signal when homodyne detection is employed for readout. We show that displacement sensitivity of the device in this region may exceed the upper bound imposed upon the sensitivity when operating in the linear region. On the other hand, we find that the high displacement sensitivity is accompanied by a slowing down of the response of the system, resulting in a limited bandwidth.

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I. INTRODUCTION

Resonant detection is a widely employed technique in a variety of applications. A detector belonging to this class typically consists of a resonator, which is characterized by a resonance frequency ω_0 and characteristic damping rates. Detection is achieved by coupling the measured physical parameter of interest, denoted as x, to the resonator in such a way that ω_0 becomes effectively x dependent, that is $\omega_0 = \omega_0(x)$. In such a configuration x can be measured by externally driving the resonator, and monitoring its response as a function of time by measuring some output signal $X(t)$. Such a scheme allows a sensitive measurement of the parameter x, provided that the average value of $X(t)$, which is denoted as X_0, strongly depends on ω_0, and provided that ω_0, in turn, strongly depends on x. These dependencies are characterized by the responsivity factors $R = |\partial X_0/\partial\omega_0|$ and $|\partial\omega_0/\partial x|$ respectively. Resonant detection has been employed before for mass detection [1], quantum state readout of a quantum bit [2, 3, 4, 5], detection of gravitational waves [6], and in many other applications.

In general, any detection scheme employed for monitoring the parameter of interest x can be characterized by two important figures of merit. The first is the minimum detectable change in x, denoted as δx. This parameter is determined by the above mentioned responsivity factors, the noise level, which is usually characterized by the spectral density of $X(t)$, and by the averaging time τ employed for measuring the output signal $X(t)$. The second figure of merit is the ring-down time t_{RD}, which is a measure of the detector's response time to a sudden change in x.

In general, the minimum detectable change δx is proportional to the square root of the available bandwidth, namely, to $(2\pi/\tau)^{1/2}$. It is thus convenient to characterize the sensitivity by the minimum detectable change δx per square root of bandwidth, which is given by $P_x = \delta x/(2\pi/\tau)^{1/2}$. Under some conditions, which will be discussed below in detail, the smallest possible value of P_x is given by [1, 7]

$$P_x^{\mathrm{SLL}} = \left|\frac{\partial\omega_0}{\partial x}\right|^{-1}\left(\frac{\omega_0}{2Q}\frac{k_BT}{U_0}\frac{\hbar\omega_0}{2k_BT}\coth\frac{\hbar\omega_0}{2k_BT}\right)^{1/2}, \quad (1)$$

where k_BT is the thermal energy, U_0 is the energy stored in the resonator, and Q is the quality factor of the resonator. One of the assumptions, which are made in order to derive Eq. (1), is that the response of the resonator is linear. We therefore refer to the value of P_x given by Eq. (1) as the *standard linear limit* (SLL) of resonant detection. Under the same conditions and assumptions, the ring-down time is given by

$$t_{\mathrm{RD}} = \frac{Q}{\omega_0}\ . \quad (2)$$

As can be seen from Eq. (1), sensitivity enhancement can be achieved by increasing Q, however, this unavoidably will be accompanied by an undesirable increase in the ring-down time (see Eq. (2)), namely, slowing down the response of the system to changes in x. Moreover, Eq. (1) apparently suggests that unlimited reduction in P_x can be achieved by increasing U_0 by means of increasing the drive amplitude. Note however that Eq. (1), which was derived by assuming the case of linear response, is not applicable in the nonlinear region. Thus, in order to characterize the performance of the system when nonlinear oscillations are excited by an intense drive, one has to generalize the analysis by taking nonlinearity into account [3, 7, 8, 9, 10, 11].

In the present paper we theoretically study a novel configuration for resonant detection of displacement of a

nanomechanical resonator [12, 13], which is coupled to both, a radio frequency superconducting interference device (RF SQUID) and to a superconducting stripline resonator. Similar configurations have been studied recently in [14, 15, 16, 17, 18, 19]. We employ an adiabatic approximation and a rotating wave approximation (RWA) to simplify the analysis and calculate the displacement sensitivity. We first consider the case where the response of the stripline resonator is linear and reproduce Eqs. (1) and (2) in this limit. However, we find that the response becomes nonlinear at a relatively low input drive power. Next, we show that P_x in the nonlinear region may become significantly smaller than the value given by Eq. (1), exceeding thus the SLL imposed upon the sensitivity when operating in the linear region. On the other hand, we find that the enhanced displacement sensitivity is accompanied by a slowing down of the response of the system, resulting in a limited bandwidth, namely, a ring-down time much longer than the value given by Eq. (2).

II. THE DEVICE

The device, which is schematically shown in Fig. 1, consists of an RF SQUID inductively coupled to a stripline resonator. The loop of the RF SQUID, which has a self inductance Λ, is interrupted by a Josephson junction (JJ) having a critical current I_c and a capacitance C_J. A perpendicularly applied magnetic field produces a flux Φ_e threading the loop of the RF SQUID. The stripline resonator is made of two identical stripline sections of length $l/2$ each having inductance L_T and capacitance C_T per unit length and characteristic impedance $Z_T = \sqrt{L_T/C_T}$. The stripline sections are connected by a doubly clamped beam made of a narrow strip, which is freely suspended and allowed to oscillate mechanically. We assume the case where the fundamental mechanical mode vibrates in the plane of the figure and denote the amplitude of this flexural mode as x. Let m be the effective mass of the fundamental mechanical mode, and ω_m its angular resonance frequency. The suspended mechanical beam is assumed to have an inductance L_b (independent of x) and a negligible capacitance to ground. The coupling between the mechanical resonator and the rest of the system originates from the dependence of the mutual inductance M between the inductor L_b and the RF SQUID loop on the mechanical displacement x, that is $M = M(x)$.

A. Transmission Line Resonator

The transmission line is assumed to extend from $y = -l/2$ to $y = l/2$, and the lumped inductance L_b is located at $y = 0$. Consider the case where only the fundamental mode of the resonator is driven. Disregarding all other modes we express the voltage $\mathcal{V}(y,t)$ and current $\mathcal{I}(y,t)$

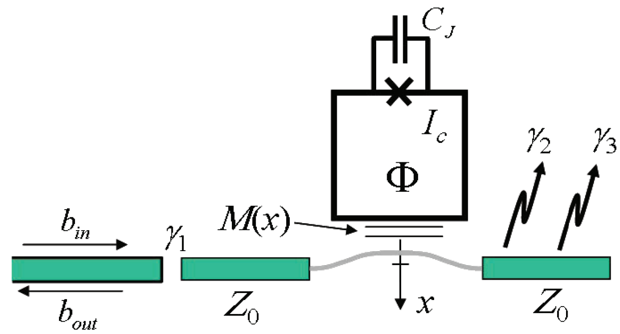

FIG. 1: (Color online) The device consists of a nanomechanical resonator coupled to both, a radio frequency superconducting interference device (RF SQUID) and to a superconducting stripline resonator.

along the transmission line as

$$\mathcal{V}(y,t) = \begin{cases} \dot{\varphi}\frac{L_T\cos\left[\kappa\left(y+\frac{l}{2}\right)\right]}{\kappa L_b \sin\frac{\kappa l}{2}} & y<0 \\ -\dot{\varphi}\frac{L_T\cos\left[\kappa\left(y-\frac{l}{2}\right)\right]}{\kappa L_b \sin\frac{\kappa l}{2}} & y>0 \end{cases} , \tag{3}$$

$$\mathcal{I}(y,t) = \begin{cases} \varphi\frac{\sin\left[\kappa\left(y+\frac{l}{2}\right)\right]}{L_b \sin\frac{\kappa l}{2}} & y<0 \\ -\varphi\frac{\sin\left[\kappa\left(y-\frac{l}{2}\right)\right]}{L_b \sin\frac{\kappa l}{2}} & y>0 \end{cases} , \tag{4}$$

where φ represents the flux in the lumped inductor at $y = 0$. The value of κ is determined by applying Faraday's law to the lumped inductor at $y = 0$ [18]

$$\cot\frac{\kappa l}{2} = -\frac{\kappa l}{2}\frac{L_b}{L_T l} . \tag{5}$$

For the fundamental mode the solution is in the range $\pi \le \kappa l \le 2\pi$ (see Fig. 2).

B. Inductive Coupling

The total magnetic flux Φ threading the loop of the RF SQUID is given by

$$\Phi = \Phi_e + \Phi_i , \tag{6}$$

where Φ_i represents the flux generated by both, the circulating current in the RF SQUID I_s and by the current in the suspended mechanical beam I_b

$$\Phi_i = I_s\Lambda + MI_b , \tag{7}$$

where Λ is the self inductance of the loop. Similarly, the magnetic flux φ in the inductor L_b is given by

$$\varphi = I_bL_b + MI_s . \tag{8}$$

Inverting these relations yields

$$I_s = \frac{L_b\Phi_i - M\varphi}{\Lambda L_b\left(1-\alpha_M^2\right)} , \tag{9}$$

$$I_b = \frac{\Lambda\varphi - M\Phi_i}{\Lambda L_b\left(1-\alpha_M^2\right)} , \tag{10}$$

where

$$\alpha_M = \frac{M}{\sqrt{\Lambda L_b}} . \tag{11}$$

The gauge invariant phase across the Josephson junction θ is given by

$$\theta = 2\pi n - \frac{2\pi\Phi}{\Phi_0} , \tag{12}$$

where n is an integer and $\Phi_0 = h/2e$ is the flux quantum. We set $n = 0$, since observable quantities do not depend on n.

C. Capacitive and Inductive Energies

Assuming that the only excited mode is the fundamental one, the capacitive energy stored in the stripline resonator is found using Eqs. (3) and (5)

$$\frac{C_T}{2} \int_{-l/2}^{l/2} \mathcal{V}^2 \, \mathrm{d}y = \frac{C_e \dot{\varphi}^2}{2} , \tag{13}$$

where C_e, which is given by

$$C_e = \frac{C_T L_T}{2\kappa^2 \vartheta L_b} , \tag{14}$$

represents the effective capacitance of the stripline resonator. The factor ϑ, which is defined by

$$\vartheta = -\frac{\sin(\kappa l)}{\kappa l + \sin(\kappa l)} , \tag{15}$$

can be calculated by solving numerically Eq. (5) (see Fig. 2). Similarly, the inductive energy stored in the resonator, excluding the energy stored in the lumped inductor L_b at $y = 0$, is found using Eqs. (4) and (5)

$$\frac{L_T}{2} \int_{-l/2}^{l/2} \mathcal{I}^2 \, \mathrm{d}y = \frac{\varphi^2}{2L_e} , \tag{16}$$

where L_e, which is given by

$$\frac{1}{L_e} = C_e \omega_e^2 + \frac{1}{L_b} , \tag{17}$$

represents the effective inductance of the stripline resonator excluding the lumped element at $y = 0$, and where

$$\omega_e = \frac{\kappa}{\sqrt{L_T C_T}} . \tag{18}$$

The inductive energy stored in the RF SQUID loop and the lumped inductor L_b is calculated using Eqs. (9) and (10)

$$\frac{1}{2} \begin{pmatrix} I_s & I_b \end{pmatrix} \begin{pmatrix} \Lambda & M \\ M & L_b \end{pmatrix} \begin{pmatrix} I_s \\ I_b \end{pmatrix} = \frac{\varphi^2}{2L_b} + \frac{\left(\Phi - \Phi_e - \frac{M\varphi}{L_b}\right)^2}{2\Lambda\left(1 - \alpha_M^2\right)} . \tag{19}$$

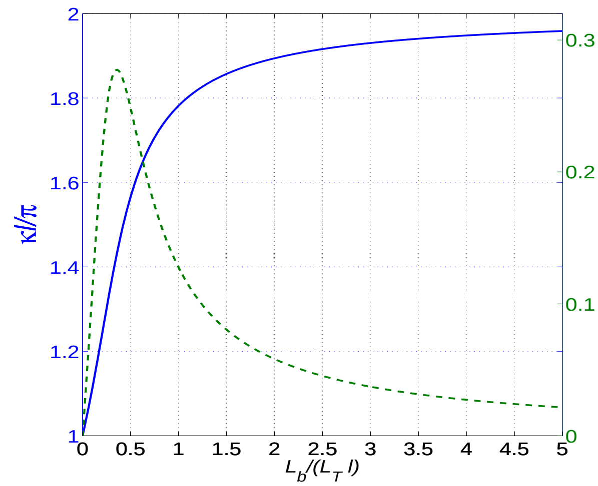

FIG. 2: (color online) The factor κl (blue solid line) and ϑ (green dashed line) calculated using Eq. (5) as a function $L_b/L_T l$.

III. LAGRANGIAN AND HAMILTONIAN OF THE CLOSED SYSTEM

Here we derive a Lagrangian for the closed system consisting of the nanomechanical resonator, stripline resonator and the RF SQUID. The effect of damping will be later taken into account by introducing coupling to thermal baths. The Lagrangian of the closed system is expressed as a function of x, φ and Φ and their time derivatives (denoted by overdot)

$$\mathcal{L} = \frac{m\dot{x}^2}{2} + \frac{C_e \dot{\varphi}^2}{2} + \frac{C_J \dot{\Phi}^2}{2} - U_0 - U_1 , \tag{20}$$

where the potential terms are given by

$$U_0 = \frac{m\omega_m^2 x^2}{2} + \frac{C_e \omega_e^2 \varphi^2}{2} , \tag{21a}$$

$$U_1 = \frac{\left(\Phi - \Phi_e - \frac{M\varphi}{L_b}\right)^2}{2\Lambda\left(1 - \alpha_M^2\right)} - \frac{\Phi_0 I_c \cos\frac{2\pi\Phi}{\Phi_0}}{2\pi} . \tag{21b}$$

Using Eqs. (6), (9), (10) and (11), the corresponding Euler - Lagrange equations can be expressed as

$$m\ddot{x} + m\omega_m^2 x - I_s I_b \frac{\mathrm{d}M}{\mathrm{d}x} = 0 , \tag{22a}$$

$$C_e \ddot{\varphi} + \frac{\varphi}{L_e} + I_b = 0 , \tag{22b}$$

$$C_J \ddot{\Phi} + I_s + I_c \sin\frac{2\pi\Phi}{\Phi_0} = 0 . \tag{22c}$$

The interpretation of these equations of motion is straightforward. Eq. (22a) is Newton's 2nd law for the mechanical resonator, where the force is composed of the restoring elastic force $-m\omega_m^2 x$ and the term due to the dependence of M on x. Eq. (22b) relates the current

in the suspended beam I_b with the currents in the effective capacitor C_e and the effective inductor L_e. Eq. (22c) states that the circulating current I_s equals the sum of the current $I_c \sin\theta$ through the JJ and the current $C_J \dot{V}_J$ through the capacitor, where the voltage V_J across the JJ is given by the second Josephson equation $V_J = (\Phi_0/2\pi)\,\dot{\theta}$.

The variables canonically conjugate to x, φ and Φ are given by $p = m\dot{x}$, $q = C_e\dot{\varphi}$ and $Q = C_J\dot{\Phi}$ respectively. The Hamiltonian is given by

$$\mathcal{H} = p\dot{x} + q\dot{\varphi} + Q\dot{\Phi} - \mathcal{L} = \mathcal{H}_0 + \mathcal{H}_1 \ , \tag{23}$$

where

$$\mathcal{H}_0 = \frac{p^2}{2m} + \frac{q^2}{2C_e} + U_0 \ , \tag{24}$$

$$\mathcal{H}_1 = \frac{Q^2}{2C_J} + U_1 \ . \tag{25}$$

Quantization is achieved by regarding the variables $\{x, p, \varphi, q, \Phi, Q\}$ as Hermitian operators satisfying Bose commutation relations.

IV. ADIABATIC APPROXIMATION

As a basis for expanding the general solution we use the eigenvectors of the following Schrödinger equation

$$\mathcal{H}_1 \left|n(x,\varphi)\right\rangle = \varepsilon_n(x,\varphi)\left|n(x,\varphi)\right\rangle \ , \tag{26}$$

where x and φ are treated here as parameters (rather than degrees of freedom). The local eigen-vectors are assumed to be orthonormal

$$\left\langle m(x,\varphi)\right|\left.n(x,\varphi)\right\rangle = \delta_{nm} \ . \tag{27}$$

The eigenenergies $\varepsilon_n(x,\varphi)$ and the associated wavefunctions φ_n are found by solving the following Schrödinger equation

$$\left(-\beta_C\frac{\partial^2}{\partial\phi^2} + u\right)\varphi_n = \frac{\varepsilon_n}{E_0}\varphi_n \ . \tag{28}$$

where

$$u = \frac{(\phi-\phi_0)^2}{1-\alpha_M^2} + 2\beta_L\cos\phi \ , \tag{29}$$

$$\phi = \frac{2\pi\Phi}{\Phi_0} - \pi \ , \tag{30}$$

$$\phi_0 = \frac{2\pi\Phi_e}{\Phi_0} + \frac{2\pi M(x)\varphi}{\Phi_0 L_b} - \pi \ , \tag{31}$$

$$\beta_L = \frac{2\pi\Lambda I_c}{\Phi_0} \ , \tag{32}$$

$$\beta_C = \frac{2e^2}{C_J E_0} \ , \tag{33}$$

$$E_0 = \frac{\Phi_0^2}{8\pi^2\Lambda} \ . \tag{34}$$

The total wave function is expanded as

$$\psi = \sum_n \xi_n(x,\varphi,t)\left|n\right\rangle \ . \tag{35}$$

In the adiabatic approximation [20] the time evolution of the coefficients ξ_n is governed by the following set of decoupled equations of motion

$$\left[\mathcal{H}_0 + \varepsilon_n(x,\varphi)\right]\xi_n = i\hbar\dot{\xi}_n \ . \tag{36}$$

Note that in the present case the geometrical vector potential [20] vanishes since the wavefunctions $\varphi_n(\phi)$ can be chosen to be real. The validity of the adiabatic approximation will be discussed below.

V. TWO LEVEL APPROXIMATION

In what follows we focus on the case where $|\phi_0| \ll 1$ and $\beta_L\left(1-\alpha_M^2\right) > 1$. In this case the adiabatic potential $u(\phi)$ given by Eq. (29) contains two wells separated by a barrier near $\phi = 0$. At low temperatures only the two lowest energy levels contribute. In this limit the local Hamiltonian $\mathcal{H}_1$ can be expressed in the basis of the states $|\curvearrowleft\rangle$ and $|\curvearrowright\rangle$, representing localized states in the left and right well respectively having opposite circulating currents. In this basis, $\mathcal{H}_1$ can be expressed using Pauli's matrices

$$\mathcal{H}_1 \doteq \eta\phi_0\sigma_z + \Delta\sigma_x \ . \tag{37}$$

The real parameters η and Δ can be determined by solving numerically the Schrödinger equation (28) [14]. The eigenvectors and eigenenergies are denoted as $\mathcal{H}_1\left|\pm\right\rangle = \varepsilon_\pm\left|\pm\right\rangle$, where

$$\varepsilon_\pm = \pm\sqrt{\eta^2\phi_0^2 + \Delta^2} \ . \tag{38}$$

VI. ROTATING WAVE APPROXIMATION

Consider the case where $\Phi_e = \Phi_0/2$, that is

$$\phi_0 = \frac{2\pi M(x)\varphi}{\Phi_0 L_b} \ , \tag{39}$$

and assume that adiabaticity holds and that the RF SQUID remains in its lowest energy state. In this case, as can be seen from Eq. (38), expansion of $\varepsilon_-(x,\varphi)$ yields only even powers of φ. These even powers of φ can be expressed in terms of the annihilation operator

$$A_e = \frac{e^{i\omega_e t}}{\sqrt{2\hbar}}\left(\sqrt{C_e\omega_e}\varphi + \frac{i}{\sqrt{C_e\omega_e}}q\right) \tag{40}$$

and its Hermitian conjugate $A_e^\dagger$, yielding terms oscillating at frequencies $2n\omega_e$, where n is integer. In the RWA such terms are neglected unless $n = 0$ since the effect

of the oscillating terms on the dynamics on a time scale much longer than a typical oscillation period is negligibly small [21]. Moreover, constant terms in the Hamiltonian are disregarded since they only give rise to a global phase factor. Displacement detection is performed by externally driving the fundamental mode of the stripline resonator. To study the effect of nonlinearity to lowest order we keep terms up to 4th order in φ. On the other hand, since the mechanical displacement is assumed to be very small, we keep terms up to 1st order only in x. Thus, in the RWA the Hamiltonian $\mathcal{H}_0+\varepsilon_-$ is given by

$$\mathcal{H}_{\mathrm{RWA}}=\hbar\omega_m N_m+\hbar\omega_0\left(x\right)N_e+\hbar K N_e^2\ , \tag{41}$$

where N_m and $N_e=A_e^\dagger A_e$ are number operators of the mechanical and stripline resonators respectively,

$$K=\frac{3\Delta}{4\hbar}\left(\frac{\eta}{\Delta}\frac{2\pi M_0}{\Phi_0 L_b}\sqrt{\frac{\hbar}{2C_e\omega_e}}\right)^4\ , \tag{42}$$

$$\omega_0\left(x\right)=\omega_e-\Omega_2\left(1+2\frac{\mathrm{d}\log M}{\mathrm{d}x}x\right)\ , \tag{43}$$

$$\Omega_2=\frac{\Delta}{\hbar}\left(\frac{\eta}{\Delta}\frac{2\pi M_0}{\Phi_0 L_b}\sqrt{\frac{\hbar}{2C_e\omega_e}}\right)^2\ , \tag{44}$$

and $M_0=M\left(0\right)$.

VII. HOMODYNE DETECTION

The stripline resonator is weakly coupled (with a coupling constant γ_1) to a semi infinite feedline, which guides the input and output RF signals. To model the effect of dissipation (both linear and nonlinear), we add two fictitious semi infinite transmission lines to the model, which allow energy escape from the resonator. The first transmission line is linearly coupled to the resonator with a coupling constant γ_2, and the second one is nonlinearly coupled with a coupling constant γ_3 [22].

The dependence of ω_0 on x can be exploited for displacement detection. This is achieved by exciting the fundamental mode of the stripline resonator by launching into the feedline a monochromatic input pump signal having a real amplitude b_{in} and an angular frequency ω_p close to the resonance frequency ω_0. The output signal c_{out} reflected off the resonator is measured using homodyne detection, which is performed by employing a balance mixing with an intense local oscillator having the same frequency as the pump frequency ω_p, and an adjustable phase ϕ_{LO}. That is, the normalized (with respect to the amplitude of the local oscillator) output signal of the homodyne detector is given by

$$X_{\phi_{\mathrm{LO}}}=c_{out}^\dagger e^{-i\phi_{\mathrm{LO}}}+c_{out}e^{i\phi_{\mathrm{LO}}}\ . \tag{45}$$

To proceed, we employ below some results of Ref. [22], which has studied a similar case of homodyne detection of a driven nonlinear resonator.

A. Equation of Motion

Using the standard method of Gardiner and Collett [23], and applying a transformation to a reference frame rotating at angular frequency ω_p

$$A_e=Ce^{-i\omega_p t}\ , \tag{46}$$

yield the following equation for the operator C

$$\frac{\mathrm{d}C}{\mathrm{d}t}+\Theta=F\left(t\right)\ , \tag{47}$$

where

$$\Theta\left(C,C^\dagger\right)=\left[\gamma+i\left(\omega_0-\omega_p\right)+\left(iK+\gamma_3\right)C^\dagger C\right]C+i\sqrt{2\gamma_1}b_{in}e^{i\phi_1}\ , \tag{48}$$

$\gamma=\gamma_1+\gamma_2$, ϕ_1 is the (real) phase shift of transmission from the feedline into the resonator, and $F\left(t\right)$ is a noise term, having a vanishing average $\langle F\left(t\right)\rangle=0$, and an autocorrelation function, which is determined by assuming that the three semi-infinite transmission lines are at thermal equilibrium at temperatures T_1, T_2 and T_3 respectively.

B. Linearization

Let $C=B+c$, where B is a complex number for which

$$\Theta\left(B,B^*\right)=0\ , \tag{49}$$

namely, B is a steady state solution of Eq. (47) for the noiseless case $F=0$. When the noise term F can be considered as small, one can find an equation of motion for the fluctuation around B by linearizing Eq. (47)

$$\frac{\mathrm{d}c}{\mathrm{d}t}+Wc+Vc^\dagger=F\ , \tag{50}$$

where

$$W=\left.\frac{\partial\Theta}{\partial C}\right|_{C=B}=\gamma+i\left(\omega_0-\omega_p\right)+2\left(iK+\gamma_3\right)B^*B\ , \tag{51}$$

and

$$V=\left.\frac{\partial\Theta}{\partial C^\dagger}\right|_{C=B}=\left(iK+\gamma_3\right)B^2\ . \tag{52}$$

C. Onset of Bistability Point

In general, for any fixed value of the driving amplitude b_{in}, Eq. (49) can be expressed as a relation between $\left|B\right|^2$ and ω_p. When b_{in} is sufficiently large the response of the

resonator becomes bistable, that is $|B|^2$ becomes a multi-valued function of ω_p. The onset of bistability point is defined as the point for which

$$\frac{\partial \omega_p}{\partial |B|^2} = 0 \ , \tag{53}$$

$$\frac{\partial^2 \omega_p}{\partial \left(|B|^2\right)^2} = 0 \ . \tag{54}$$

Such a point occurs only if the nonlinear damping is sufficiently small [22], namely, only when the following condition holds

$$|K| > \sqrt{3}\gamma_3 \ . \tag{55}$$

At the onset of bistability point the drive frequency and amplitude are given by

$$(\omega_p - \omega_0)_c = \gamma \frac{K}{|K|} \left[\frac{4\gamma_3 |K| + \sqrt{3}\left(K^2 + \gamma_3^2\right)}{K^2 - 3\gamma_3^2} \right] \ , \tag{56}$$

$$(b_{in})_c = \frac{8}{3\sqrt{3}} \frac{\gamma^3 (K^2 + \gamma_3^2)}{\left(|K| - \sqrt{3}\gamma_3\right)^3} \ , \tag{57}$$

and the resonator mode amplitude is

$$|B|_c^2 = \frac{2\gamma}{\sqrt{3}\left(|K| - \sqrt{3}\gamma_3\right)} \ . \tag{58}$$

D. Ring-Down Time

The solution of the equation of motion (50) can be expressed as [22]

$$c(t) = \int_{-\infty}^{\infty} \mathrm{d}t' G(t - t') \Gamma(t') \ , \tag{59}$$

where

$$\Gamma(t) = \frac{\mathrm{d}F(t)}{dt} + W^* F(t) - V F^\dagger(t) \ . \tag{60}$$

The propagator is given by

$$G(t) = u(t) \frac{e^{-\lambda_0 t} - e^{\lambda_1 t}}{\lambda_1 - \lambda_0} \ , \tag{61}$$

where $u(t)$ is the unit step function, and the Lyapunov exponents λ_0 and λ_1 are the eigenvalues of the homogeneous equation, which satisfy

$$\lambda_0 + \lambda_1 = 2W' \ , \tag{62}$$

$$\lambda_0 \lambda_1 = |W|^2 - |V|^2 \ , \tag{63}$$

where W' is the real part of W. Thus one has

$$\lambda_{0,1} = W' \left(1 \pm \sqrt{1 + \frac{|W|^2}{(W')^2} (\zeta^2 - 1)} \right) \ , \tag{64}$$

where

$$\zeta = \left| \frac{V}{W} \right| \ . \tag{65}$$

We chose to characterize the ring-down time scale as

$$t_{\mathrm{RD}} = \lambda_0^{-1} + \lambda_1^{-1} = \frac{2W'}{|W|^2 (1 - \zeta^2)} \ . \tag{66}$$

Note that in the limit $\zeta \to 1$ slowing down occurs and $t_{\mathrm{RD}} \to \infty$. This limit corresponds to the case of operating the resonator near a jump point, close to the edge of the bistability region. On the other hand, the limit $\zeta = 0$ corresponds to the linear case, for which t_{RD} at resonance ($\omega_p = \omega_0$) is given by Eq. (2).

VIII. DISPLACEMENT SENSITIVITY

Consider a measurement in which $X_{\phi_{\mathrm{LO}}}(t)$ is monitored in the time interval $[0, \tau]$, and the average measured value is used to estimate the displacement x. Assuming the case where τ is much longer than the characteristic correlation time of $X_{\phi_{\mathrm{LO}}}(t)$, one finds that the minimum detectable displacement is given by [1]

$$\delta x = \left| \frac{\partial X_0}{\partial x} \right|^{-1} \left(\frac{2\pi}{\tau} \right)^{1/2} P_X^{1/2}(0) \ . \tag{67}$$

Moreover, using Eq. (43), one finds the minimum detectable displacement per square root of bandwidth, which is defined as $P_x = (\tau/2\pi)^{1/2} \delta x$, is given by

$$P_x = \left| 2\Omega_2 \frac{\mathrm{d}\log M}{\mathrm{d}x} \right|^{-1} R^{-1} P_X^{1/2}(0) \ , \tag{68}$$

where R is the responsivity with respect to a change in ω_0, namely $R = |\partial X_0 / \partial \omega_0|$. To evaluate P_x we calculate below the responsivity R and the spectral density $P_X^{1/2}(0)$ using results, which were obtained in Ref. [22].

A. Responsivity

In steady state, when input noise is disregarded, the amplitude of the fundamental mode of the stripline resonator is expressed as $Be^{-i\omega_p t}$, where the complex number B is found by solving Eq. (49), and the average output signal is expressed as $b_{out} e^{-i\omega_p t}$ (b_{out} is in general complex). The amplitude of the output signal in the feedline b_{out} is related to the input signal b_{in} and the mode amplitude B by an input-output relation [23]

$$b_{out} = b_{in} - i\sqrt{2\gamma_1} e^{-i\phi_1} B \ . \tag{69}$$

Consider a small change in the resonance frequency $\delta\omega_0$. The resultant change in the steady state mode amplitude δB can be calculated using Eq. (49)

$$i\left(\delta\omega_0\right)B+W\delta B+V\left(\delta B\right)^*=0\;. \tag{70}$$

The solution of this equation together with Eqs. (45) and (69) allows calculating the responsivity

$$R=\frac{2\sqrt{2\gamma_1}\left|B\right|}{\left|W\right|\left(1-\zeta^2\right)}\left|\sin\left(\phi_t+\phi_C\right)+\zeta\sin\left(\phi_t-\phi_C\right)\right|\;, \tag{71}$$

where

$$\phi_t=\frac{2\phi_{\mathrm{LO}}-\phi_W+\phi_V-2\phi_1}{2}\;, \tag{72}$$

$$\phi_C=\frac{2\phi_B-\phi_W-\phi_V-\pi}{2}\;, \tag{73}$$

$e^{i\phi_B}=B/\left|B\right|$, $e^{i\phi_W}=W/\left|W\right|$ and $e^{i\phi_V}=V/\left|V\right|$.

B. Spectral Density

The zero frequency spectral density $P_X\left(0\right)$ was calculated in Ref. [22]

$$\begin{aligned}P_X\left(0\right)=&\left|\frac{2\gamma_1\left(1-\zeta e^{2i\phi_t}\right)}{\left|W\right|\left(1-\zeta^2\right)}-e^{-i\phi_W}\right|^2\coth\frac{\hbar\omega_0}{2k_BT_1}\\&+\frac{\gamma_2}{\gamma_1}\left|\frac{2\gamma_1\left(1-\zeta e^{2i\phi_t}\right)}{\left|W\right|\left(1-\zeta^2\right)}\right|^2\coth\frac{\hbar\omega_0}{2k_BT_2}\\&+\frac{2\gamma_3\left|B\right|^2}{\gamma_1}\left|\frac{2\gamma_1\left(1-\zeta e^{2i\phi_t}\right)}{\left|W\right|\left(1-\zeta^2\right)}\right|^2\coth\frac{\hbar\omega_0}{2k_BT_3}\;.\end{aligned} \tag{74}$$

C. The Linear Case

In this case $K=\gamma_3=0$, $W=i\left(\omega_0-\omega_p\right)+\gamma$ and $V=0$, and consequently $\zeta=0$. Thus, the responsivity is given by

$$R=\frac{2\sqrt{2\gamma_1}\left|B\right|\left|\sin\left(\phi_t+\phi_C\right)\right|}{\left|W\right|}\;. \tag{75}$$

Moreover, for the case where $T_1=T_2$, Eq. (74) becomes

$$P_X\left(0\right)=\coth\frac{\hbar\omega_0}{2k_BT_1}\;. \tag{76}$$

The largest responsivity is obtained at resonance, namely when $\omega_p=\omega_0$, and when the homodyne detector measures the phase of oscillations, namely, when $\left|\sin\left(\phi_t+\phi_C\right)\right|=1$. For this case P_x obtains its smallest value, which is denoted as P_{x0}, and is given by

$$P_{x0}=\left|2\Omega_2\frac{\mathrm{d}\log M}{\mathrm{d}x}\right|^{-1}\left(\frac{\gamma^2}{8\gamma_1\left|B\right|^2}\coth\frac{\hbar\omega_0}{2k_BT_1}\right)^{1/2}\;. \tag{77}$$

Note that this result coincides with the SLL value P_x^{SLL} given by Eq. (1) provided that the quality factor in Eq. (1) is taken to be given by $Q=2\omega_0\gamma_1/\gamma^2$. That is, $P_{x0}\simeq P_x^{\mathrm{SLL}}$ in the limit of strongly overcoupled resonator, namely when the damping of the resonator is dominated by the coupling to the feedline ($\gamma_1\simeq\gamma$).

The response of the stripline resonator is approximately linear only when $\left|B\right|$ is much smaller than the critical value corresponding to the onset of nonlinear bistability. Using this critical value $\left|B\right|_c$, which is given by Eq. (58), and using Eqs. (42) and (44), one finds that the smallest possible value of P_{x0} in this regime is roughly given by

$$P_{x0,c}\simeq\frac{0.14\left|\frac{\mathrm{d}\log M}{\mathrm{d}x}\right|^{-1}}{\sqrt{\Delta/\hbar}}\sqrt{\frac{\gamma}{\gamma_1}\left(1-\frac{\sqrt{3}\gamma_3}{\left|K\right|}\right)\coth\frac{\hbar\omega_0}{2k_BT_1}}\;. \tag{78}$$

D. The General Case

The minimum detectable displacement per square root of bandwidth in the general case can be written as

$$P_x=P_x^{\mathrm{SLL}}g\left(\phi_t\right)\;, \tag{79}$$

where

$$g\left(\phi_t\right)=\frac{\frac{\left|W\right|}{\gamma}\left[\frac{\left(1-\zeta^2\right)^2P_X\left(0\right)}{\coth\frac{\hbar\omega_0}{2k_BT_1}}\right]^{1/2}}{\left|\sin\left(\phi_t+\phi_C\right)+\zeta\sin\left(\phi_t-\phi_C\right)\right|}\;, \tag{80}$$

and $P_X\left(0\right)$ is given by Eq. (74). The function $g\left(\phi_t\right)$ is periodic with a period π. Its minimum value is denoted as $g_{\min}$. Beating the SLL given by Eq. (1) is achieved when $g_{\min}$ is made smaller than unity.

IX. VALIDITY OF APPROXIMATIONS

In this section we examine the conditions, which are required to justify the approximations made, and determine the range of validity of our results. Clearly, our analysis breaks down if the driving amplitude b_{in} is made sufficiently large. In this case both the adiabatic approximation and the assumption that back-reaction effects are negligibly small will become invalid. The range of validity of the adiabatic approximation is examined below by estimating the rate of Zener transitions. Moreover, we study below the conditions under which back-reaction effects

acting back on the mechanical resonator play an important role. Using these results we derive conditions for the validity of the above mentioned approximations. These conditions are then examined for the case where the system is driven to the onset of nonlinear bistability. This analysis allows us to determine whether the device can be operated in the regime of nonlinear bistability, where the effects of bifurcation amplification [24, 25, 26, 27, 28, 29] and noise squeezing can be exploited [22, 30, 31, 32], without, however, violating the adiabatic approximation and without inducing strong back-reaction effects.

A. Adiabatic Condition

As before, consider the case where the externally applied flux is given by $\Phi_e = \Phi_0/2$, and the stripline resonator is driven close to the onset of nonlinear bistability, where the number of photons approaches the critical value given by Eq. (58), and assume for simplicity the case where $\gamma_3 \ll |K|$. Using Refs. [14, 33] one finds that adiabaticity holds, namely Zener transitions between adiabatic states are unlikely, provided that

$$\frac{\pi\Delta^2}{\eta\hbar\Gamma_c} \gtrsim 1 \ , \tag{81}$$

where

$$\Gamma_c = \frac{2\pi M_0}{\Phi_0 L_b}\sqrt{\frac{2\hbar\omega_e \left|B\right|_c^2}{C_e}} \ . \tag{82}$$

B. Back Reaction

Back reaction of the driven stripline resonator results in a force noise acting on the mechanical resonator and a renormalization of the mechanical resonance frequency ω_m and the damping rate γ_m [34, 35]. The renormalized values, denoted as ω_m^{eff} and γ_m^{eff}, are expressed using the renormalization factors R_{f} and R_{d}

$$R_{\mathrm{f}} = \frac{\omega_m^{\mathrm{eff}} - \omega_m}{\gamma_m} \ , \tag{83a}$$

$$R_{\mathrm{d}} = \frac{\gamma_m^{\mathrm{eff}} - \gamma_m}{\gamma_m} \ , \tag{83b}$$

which where calculated in Ref. [18]

$$R_{\mathrm{f}} = \chi_{\mathrm{f}} Q_m R_0 \ , \tag{84a}$$

$$R_{\mathrm{d}} = \chi_{\mathrm{d}} Q_m R_0 \ , \tag{84b}$$

where $Q_m = \omega_m/\gamma_m$,

$$R_0 = \frac{8\hbar\Omega_2^2 \left|B\right|^2 \left(\frac{\mathrm{d}\log M}{\mathrm{d}x}\right)^2}{m\omega_m^3} \ , \tag{85}$$

$$\chi_{\mathrm{f}} = \frac{\frac{1}{2}\frac{\Delta\omega}{\omega_m}\left[\left(\frac{\gamma}{\omega_m}\right)^2 - 1 + \left(\frac{\Delta\omega}{\omega_m}\right)^2\right]}{\left[\left(\frac{\gamma}{\omega_m}\right)^2 + \left(1+\frac{\Delta\omega}{\omega_m}\right)^2\right]\left[\left(\frac{\gamma}{\omega_m}\right)^2 + \left(1-\frac{\Delta\omega}{\omega_m}\right)^2\right]} \ , \tag{86a}$$

$$\chi_{\mathrm{d}} = -\frac{\frac{\gamma}{\omega_m}\frac{\Delta\omega}{\omega_m}}{\left[\left(\frac{\gamma}{\omega_m}\right)^2 + \left(1+\frac{\Delta\omega}{\omega_m}\right)^2\right]\left[\left(\frac{\gamma}{\omega_m}\right)^2 + \left(1-\frac{\Delta\omega}{\omega_m}\right)^2\right]} \ , \tag{86b}$$

and

$$\Delta\omega = \omega_e - \Omega_2 - \omega_p \ . \tag{87}$$

We now wish to examine whether backreaction effects are important when the stripline resonator is driven to the onset of nonlinear bistability. For simplicity we neglect nonlinear damping, that is we take $\gamma_3 = 0$. Using Eqs. (42), (44) and (58), one finds that the value of R_0 [Eq. (85)] at the onset of nonlinear bistability, which is denoted as R_{0c}, is given by

$$R_{0c} = \frac{\gamma}{\omega_m}\frac{64\Delta\left(\frac{\mathrm{d}\log M}{\mathrm{d}x}\right)^2}{3\sqrt{3}m\omega_m^2} \ . \tag{88}$$

It is straightforward to show that $|\chi_{\mathrm{f}}| < \omega_m/\gamma$ and $|\chi_{\mathrm{d}}| < \omega_m/\gamma$ for any value of $\Delta\omega$. Thus, back-reaction can be considered as negligible when

$$\frac{\omega_m^2}{\gamma\gamma_m} R_{0c} \ll 1 \ . \tag{89}$$

X. EXAMPLE

We examine below an example of a device having the following parameters: $Z_T = 50\,\Omega$, $L_b/L_T l = 1$, $\omega_e/2\pi = 5\,\mathrm{GHz}$, $\omega_e/\gamma = 10^4$, $\gamma_2 = 0.1\gamma_1$, and $\gamma_3 = 0.1K/\sqrt{3}$ for the stripline resonator, $m = 10^{-16}\,\mathrm{kg}$, $\omega_m/2\pi = 0.01\,\mathrm{GHz}$, and $\omega_m/\gamma_m = 10^4$ for the nanomechanical resonator, $\Lambda = 9.1\times10^{-10}\,\mathrm{H}$, $C_J = 0.86\times10^{-14}\,\mathrm{F}$, and $I_c = 0.44\,\mu\mathrm{A}$ for the RF SQUID, coupling terms $M_0/L_b = 0.005$, $L_b/\Lambda = 0.25$, $|\mathrm{d}\log M/\mathrm{d}x|^{-1} = 100\,\mathrm{nm}$, and temperature $T = 0.02\,\mathrm{K}$. These parameters for the stripline resonator [36], for the nanomechanical resonator [37], and for the RF SQUID [38, 39], are within reach with present day technology.

Using FastHenry simulation program (www.fastfieldsolvers.com) we find that the chosen value of Λ corresponds, for example, to a loop made of Al (penetration depth 50 nm), having a square shape with edge length of 190 μm, line width of 1 μm and film thickness of 50 nm. The values of C_J and I_c correspond to a junction having a plasma frequency of about 25 GHz.

Using these values one finds $\beta_L = 1.2$, $\beta_C = 0.1$, and $E_0/\hbar = 560\,\mathrm{GHz}$. The values of β_L, β_C and α_M are employed for calculating numerically the eigenstates of Eq. (28) [14]. From these results one finds the parameters in the two-level approximation of Hamiltonian $\mathcal{H}_1$ [Eq. (37)] $\eta = 1.2E_0$ and $\Delta = 0.073E_0$, and the nonlinear terms $\Omega_2/2\pi = 1.3\,\mathrm{MHz}$ and $K/2\pi = 0.19\,\mathrm{kHz}$.

The validity of the adiabatic approximation is confirmed by evaluating the factor in inequality (81)

$$\frac{\pi\Delta^2}{\eta\hbar\Gamma_c} = 12 \;, \tag{90}$$

whereas, to confirm that back-reaction effects can be considered as negligible we evaluate the factor in inequality (89)

$$\frac{\omega_m^2}{\gamma\gamma_m} R_{0c} = 1.4 \times 10^{-4} \;. \tag{91}$$

Using Eq. (78) we calculate the smallest possible value of P_{x0}

$$P_{x0,c} = 6.9 \times 10^{-14} \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}} \;. \tag{92}$$

A significant sensitivity enhancement, however, can be achieved by exploiting nonlinearity. Figure (3) shows the dependence of the mode amplitude $|B|$, the factor ζ, the reflection coefficient $|b_{out}/b_{in}|^2$, the enhancement factor $g_{\min}$, and the ring-down time t_{RD} on the pump frequency. Panel (a) represents the nearly linear case for which $b_{in} = 0.001\,(b_{in})_c$, panel (b) the critical case for which $b_{in} = (b_{in})_c$, and panel (c) the over-critical case for which $b_{in} = 1.5\,(b_{in})_c$. As can be clearly seen from Fig. (3) a significant sensitivity enhancement $g_{\min} \ll 1$ can be achieved when driving the resonator close to a jump point. However, in the same region, slowing down occurs, resulting in a long ring-down time $t_{\mathrm{RD}} \gg 2/\gamma$.

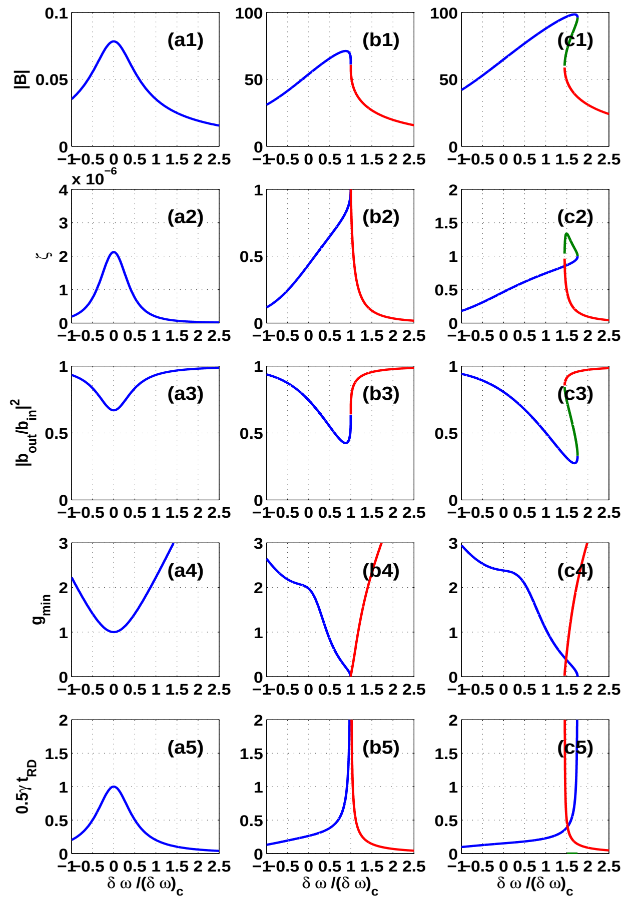

FIG. 3: (Color online) Dependence of the mode amplitude $|B|$, the factor ζ, the reflection coefficient $|b_{out}/b_{in}|^2$, the enhancement factor $g_{\min}$, and the ring-down time t_{RD} on the pump frequency. Panel (a) represents the nearly linear case for which $b_{in} = 0.001\,(b_{in})_c$, panel (b) the critical case for which $b_{in} = (b_{in})_c$, and panel (c) the over-critical case for which $b_{in} = 1.5\,(b_{in})_c$.

XI. DISCUSSION AND CONCLUSIONS

In the present paper we consider a resonant detection configuration, in which the response of the resonator is measured by monitoring an output signal in the feedline, which is reflected off the resonator. We show that operating in the regime of nonlinear response may allow a significant enhancement in the sensitivity by driving the stripline resonator near a jump point at the edge of the region of bistability. The factor $g_{\min}$, which represents this enhancement, can be made significantly smaller than unity in this limit. However, this result is not general to all resonance detectors. Consider an alternative configuration, in which, instead of measuring an output signal reflected off the resonator, the response is measured by directly homodyning the mode amplitude in the resonator. In the latter case, a similar analysis yields that the enhancement factor $g_{\min}$ cannot be made smaller than 0.5 [40]. Thus, to take full advantage of nonlinearity, it is advisable to monitor the response of the resonator by measuring a reflected off signal, rather than measuring an internal cavity signal.

As was discussed above, the largest sensitivity enhancement is obtained close to the edge of the region of bistability, where $1-\zeta \ll 1$. On the other hand, in the very same region, the perturbative approach, which we employ to study the response of the stripline resonator, becomes invalid since the fluctuation around steady state, which is assumed to be small, is strongly enhanced due to bifurcation amplification of input noise [22]. The integrated spectral density of the fluctuation, which was calculated in Eq. (80) of Ref. [40] (for the case $T_1 = T_2 = T_3$), can be employed to determine the range of validity of the perturbative approach

$$\frac{1}{1-\zeta} \coth \frac{\hbar\omega_0}{2k_B T_1} \ll |B|^2 \;. \tag{93}$$

By applying this condition to the case of operating at the onset of bistability point [see panel (b) of Fig. (3)], one finds that the smallest value of $g_{\min}$ in the region where inequality (93) holds is $\simeq 10^{-3}$ for the set of parameters chosen in the above considered example . Thus a significant sensitivity enhancement is achieved even when the region close to the jump, where the perturbative analysis breaks down, is excluded. On the other hand, the perturbative analysis cannot answer the question what is the smallest possible value of $g_{\min}$. Further work is required in order to answer this question and to properly describe the system very close to the edge of the region of bistability, where nonlinear terms of higher orders become important.

In the present paper only the case where back-reaction effects do not play an important role is considered. Thus our results are inapplicable when the displacement sensitivity approaches the value corresponding to zero point motion of the mechanical resonator $\sqrt{\hbar/m\omega_m\gamma_m}$ ($1.6\times 10^{-15}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$ for the above considered example), and consequently quantum back-reaction becomes important [41]. Moreover, the results are valid only when inequality (89) is satisfied. To increase the range of applicability of the theory and to study back-reaction effects, a more general approach will be considered in a forthcoming paper [42].

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